1. A curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = -3x^2 + 12x + 8$$

(a) Write $\mathrm{f}(x)$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The curve C has a maximum turning point at M.

(b) Find the coordinates of M.

(2)

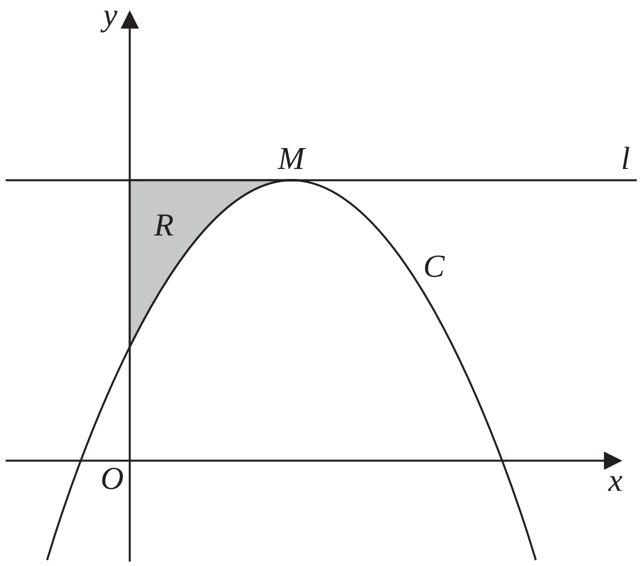


Figure 3

Figure 3 shows a sketch of the curve C.

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(c) Using algebraic integration, find the area of R.

(5)

a) $\mathrm{f}(x) = -3x^2 + 12x + 8$

$= -3(x^2 - 4x) + 8$ (1)

$= -3\left[(x-2)^2 - 4\right] + 8$ (1)

$= -3(x-2)^2 + 20$ (1)

b) By using formula from (a),

when $x = 2$, $y = 20$.

so, $M = (2, 20)$ (1)

c) The line l is $y = 20$. The area under the line l between the y-axis and M is $20 \times 2 = 40$.

So, the area of R is 40 − area under the curve C between $x = 0$ and $x = 2$.

$$R = 40 - \int_0^2 -3x^2 + 12x + 8 \, dx \quad (1)$$

$$R = 40 - (1)\left[-x^3 + 6x^2 + 8x\right]_0^2 \quad (1)$$

$$R = 40 - \left(-2^3 + 6(2)^2 + 8(2)\right) \quad (1)$$

$$R = 40 - 32 = 8 \quad (1)$$

2.

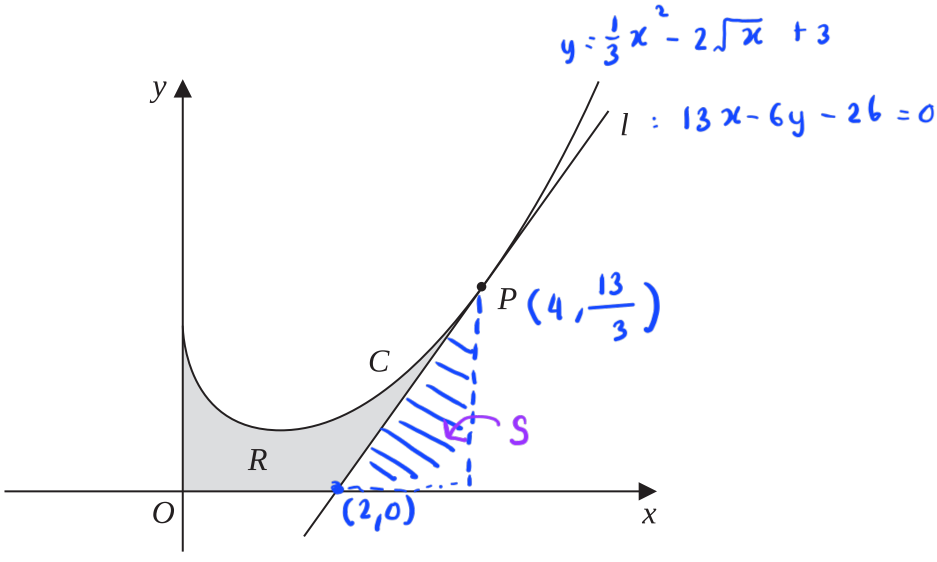


Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \qquad x \geqslant 0$$

The point P lies on C and has x coordinate 4

The line l is the tangent to C at P.

(a) Show that l has equation

$$13x - 6y - 26 = 0$$

(5)

The region R, shown shaded in Figure 2, is bounded by the y-axis, the curve C, the line l and the x-axis.

(b) Find the exact area of R.

(5)

a) $y = \frac{1}{3}x^2 - 2x^{\frac{1}{2}} + 3$

$\frac{dy}{dx} = \frac{2}{3}x - x^{-\frac{1}{2}}$ (1)

when $x = 4$, $y = \frac{1}{3} \times 4^2 - 2 \times 4^{\frac{1}{2}} + 3 = \frac{13}{3}$ $\therefore P(4, \frac{13}{3})$

$\frac{dy}{dx} = \frac{2}{3} \times 4 - 4^{-\frac{1}{2}} = \frac{13}{6}$ (1) (gradient of tangent)

Finding equation of line l:

$P(4, \frac{13}{3})$: $y - \frac{13}{3} = \frac{13}{6}(x-4)$

$6y - 2 \times 13 = 13(x-4)$

$6y - 26 = 13x - 52$

$\therefore\ l : 13x - 6y - 26 = 0$ (1)

b) Finding the x-intercept of line l:

when $y = 0$, $13x - 0 - 26 = 0$

$13x = 26 \longrightarrow \therefore x = 2$ (1)

Finding area under curve:

$$\int_0^4 \left(\frac{1}{3}x^2 - 2x^{\frac{1}{2}} + 3\right) dx$$

$$= \left[\frac{1}{9}x^3 - \frac{4}{3}x^{\frac{3}{2}} + 3x\right]_0^4 \quad (2)$$

$$= \left\{\frac{1}{9}(4)^3 - \frac{4}{3}(4)^{\frac{3}{2}} + 3(4)\right\} - \left\{0 - 0 + 0\right\}$$

$$= \frac{76}{9}$$

Finding area of triangle S :

$\frac{1}{2} \times 2 \times \frac{13}{3}$ (1)

$= \frac{13}{3}$

$\therefore$ Area of R = area under curve − area of triangle S

$= \frac{76}{9} - \frac{13}{3}$

$= \frac{37}{9}$ (1)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

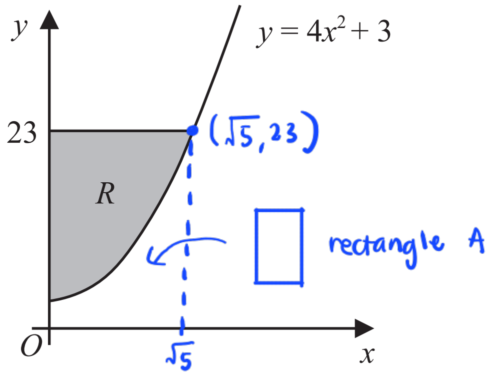


Figure 2

The finite region R, shown shaded in Figure 2, is bounded by the curve with equation $y = 4x^2 + 3$, the y-axis and the line with equation $y = 23$

Show that the exact area of R is $k\sqrt{5}$ where k is a rational constant to be found.

(5)

Finding intersection point between the curve and line y = 23 :

$23 = 4x^2 + 3$

$x^2 = 5$

$x = \sqrt{5}$ (1)

intersection point is $(\sqrt{5}, 23)$

Finding area bounded by the curve :

$\int_0^{\sqrt{5}} 4x^2 + 3 = \left[\frac{4}{3}x^3 + 3x \right]_0^{\sqrt{5}}$ (1)

$= \frac{4}{3}(\sqrt{5})^3 + 3\sqrt{5}$ (1)

Finding area of rectangle A :

$A = 23 \times \sqrt{5} = 23\sqrt{5}$

Finding area of shaded region R :

= Area of rectangle A - area bounded by the curve

$= 23\sqrt{5} - \left(\frac{4}{3}(\sqrt{5})^3 + 3\sqrt{5}\right)$ (1)

$= \frac{40}{3}\sqrt{5}$ (1)

4. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

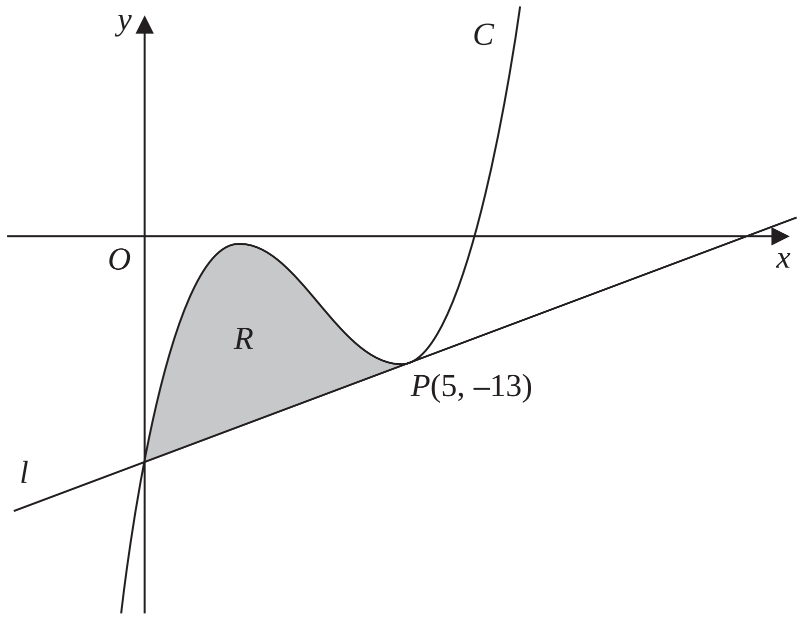


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C

The line l is the tangent to C at P

(a) Use differentiation to find the equation of l, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

(b) Hence verify that l meets C again on the y-axis.

(1)

The finite region R, shown shaded in Figure 2, is bounded by the curve C and the line l.

(c) Use algebraic integration to find the exact area of R.

(4)

a) $y = x^3 - 10x^2 + 27x - 23$

$\frac{dy}{dx} = 3x^2 - 20x + 27$ 1

$\left.\frac{dy}{dx}\right|_{x=5} = 3(5)^2 - 20(5) + 27 = 2$ 1

$\therefore$ l has gradient 2 and goes through $(5, -13)$

$\Rightarrow$ $y + 13 = 2(x - 5)$ 1

$y + 13 = 2x - 10$

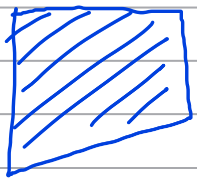
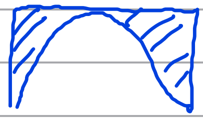
$y = 2x - 23$ 1

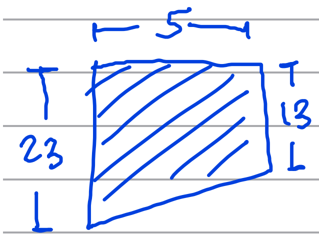
b) on the y-axis, $x=0$

$C: \; y = 0^3 - 10(0)^2 + 27(0) - 23 = -23$

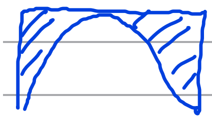
$L: \; y = 2 \times 0 - 23 = -23$

Both C and l pass through $(0,-23)$
So C meets l again on the y-axis (1)

c) Area R =  − 



$$\text{Area} = \frac{13+23}{2} \times 5 = 90 \quad (1)$$



- at the front since this area is below the x-axis, it will be negative. We are interested in the positive area.

$$\text{Area} = -\int_0^5 (x^3 - 10x^2 + 27x - 23)\,dx$$

$$= -\left[\frac{1}{4}x^4 - \frac{10}{3}x^3 + \frac{27}{2}x^2 - 23x\right]_0^5 \quad (1)$$

$$= \frac{455}{12} \quad (1)$$

$$R = 90 - \frac{455}{12} = \frac{625}{12} \quad (1)$$